J. Myanmar Acad. Arts Sci. 2020 **Vol. XVIII.No.2A**

INVESTIGATIO NO NPOLARIZATION OF LIGHT IN THE PRESENCE OF A WEAK EXTERNAL MAGNETIC FIELD

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Abstract

Degree of polarization of scattered line radiation in the presence of the magnetic field was investigated. The Stokes parameters of the scattered radiation incident on an arbitrary polarized atom in any given arbitrary direction were calculated. It was considered that the polarization of scattered line radiation from an upper atomic level with angular momentum $J_u = 3/2$ to a lower atomic level with angular momentum $J_l = 1/2$ transition. This line has transition of both type electric dipole transition and magnetic quadrupole transition. Only the electric dipole transition was considered for upper and lower atomic levels having different parities. The degree of linear polarization of scattered radiation in the presence of weak external magnetic field was calculated. In this calculation, the lower level polarization was neglected. The obtained results were compared with experimental results as well as theoretical results. It was found that the obtained results agree well with these results.

Keywords: Stokes parameters, polarization, scattered radiation, electric dipole, weak external magnetic field

Introduction

Level-crossing spectroscopy is a technique which exploits the interference phenomena that can occur in resonance fluorescence when two or more energy levels are nearly degenerate. So far it has been applied exclusively to resonance light scattering from atomic systems. When an atom is placed in the presence of magnetic field, the energy level of an atom is splited into several magnetic sublevels called Zeeman levels. If the strength of the magnetic field is strong, the Zeeman levels of an excited atomic state are distinct (i.e., separated by several natural line widths), their contribution to the atomic resonance fluorescence may be treated independently by summing over the each excited Zeeman level. When a magnetic field is applied along an arbitrary direction to this excited state, the degeneracy between the magnetic sublevels is removed. The coherence, in general, is partially destroyed and the resonance fluorescence is depolarized.

Polarization that is produced by coherent scattering modified by a weak magnetic field is known as the Hanle effect. Moreover, measuring the amount of polarization nature of the scattered radiation, we can predict the magnetic field strengths and its orientations. The Hanle effect is usually used to determine weak magnetic fields in the solar atmosphere.

Polarization is an important property of electromagnetic waves. In communications, completely polarized waves are used. The complete polarization types of electromagnetic waves are linear polarization, circular polarization and elliptical polarization. Electromagnetic waves from of ratio astronomical sources may possess random polarization (also known as un-polarized waves), partial polarization (completely polarized and un-polarized).

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Polarization of Atom and Radiation

Polarized Radiation

Polarization is a property of waves that describes the orientation of their oscillation. By convention, the polarization of light is described by specifying the direction of the wave's electric field. For transverse waves such as electromagnetic waves, it describes the orientation of the oscillations in the plane perpendicular to the wave's direction of travel. In this case, the electric field may be oriented in a single direction (linear polarization) or it may rotate as the wave travels (circular or elliptical polarization).

According to Maxwell's equations, the electric and magnetic field be perpendicular to the direction of propagation and to each other. In general, the electric field vector $\vec{E}$ can always be resolved into two perpendicular components.

If the light is linearly polarized, then the two perpendicular components with equal amplitude oscillate in phase.

Elliptically polarized light consists of two perpendicular components of unequal amplitude which differ in phase by 90 degree.

Circularly polarized light is a special case of elliptically polarized light in which the two components have same amplitude and a 90 degree phase difference. For circularly polarized light the orientation of the electric field rotates around the direction of travel. While looking at source, the electric vector of the light coming toward you appears to be rotating counterclockwise, the light is said to be right-circularly polarized. If clockwise, the light is said to be left circularly polarized light.

For unpolarized light, the electric field vector vibrates in all directions perpendicular to the direction of propagation.

Radiation Density Matrix

When the average properties of an ensemble of identical non-interacting systems are of interest, and information on the individual members of the ensemble is not needed, it is useful to introduce the concept of the density matrix. It's a matrix which describes the state of ensemble. Consider first a system in a state $|\psi\rangle$. The state $|\psi\rangle$ can be written in terms of complete orthogonal set of basis states as

$$|\psi\rangle = \sum_n C_n |u_n\rangle \tag{1}$$

$$\langle u_{n'}|\psi\rangle = \sum_n C_n \langle u_{n'}|u_n\rangle$$

$$= C_n\langle u_{n'}|u_n\rangle + C_{n'}\langle u_{n'}|u_{n'}\rangle + C_{n''}\langle u_{n'}|u_{n''}\rangle + \ldots$$

$$\langle u_{n'}|\psi\rangle = C_n \qquad (n' = n)$$

$$C_n = \langle u_{n'}|\psi\rangle$$

$|\psi\rangle$ is normalized,

$$\langle\psi\,|\,\psi\rangle = \langle\psi\,|\sum_n u_n\rangle\langle u_n\,|\,\psi\rangle$$

$$\langle\psi\,|\,\psi\rangle = \sum_n \langle\psi\,|\,u_n\rangle\langle u_n\,|\,\psi\rangle$$

$$\langle\psi\,|\,\psi\rangle = \sum_n |\,C_n\,|^2 = 1 \qquad (2)$$

An observable, such as momentum and spin components can be represented by an operator, such as A, in the vector space in question. Quite generally, an operator outs an a ket from the left,

$$A\cdot(|\,\alpha\rangle) = A\,|\,\alpha\rangle$$

If A is an observable, with matrix elements

$$\langle u_n\,|\,A\,|\,u_p\rangle = A_{np} \qquad (3)$$

The mean value of A is

$$\langle A\rangle = \langle\psi\,|\,A\,|\,\psi\rangle$$

$$\langle A\rangle = \sum_n C_n^* \sum_p C_p \langle u_n\,|\,A\,|\,u_p\rangle$$

$$\langle A\rangle = \sum_n \sum_p C_n^* C_p A_{np} \qquad (4)$$

$$\langle u_p\,|\,\psi(+)\rangle\langle\psi(+)\,|\,u_n\rangle = C_n^* C_p \qquad (5)$$

$$|\,u_{n'}\rangle\langle u_{n'}\,|\cdot|\,\psi\rangle = |\,u_{n'}\rangle\langle u_{n'}\,|\,\psi\rangle = C_{n'}\,|\,u_n\rangle$$

We see that $|\,u_{n'}\rangle\langle u_{n'}\,|$ selects that portion of the ket $|\,\psi\rangle$ parallel to $|\,u_n\rangle$. So $|\,u_{n'}\rangle\langle u_{n'}\,|$ is known as the projection operator along the base ket $|\,u_{n'}\rangle$ and it denoted by Λ.

$$\Lambda = |\,u_{n'}\rangle\langle u_{n'}\,| \qquad (6)$$

It is therefore natural to introduce the density operator ρ, defined by

$$\rho = |\,\psi\rangle\langle\psi\,| \qquad (7)$$

The density operator is represented in the $\{|\,u_n\rangle\}$ basics by a matrix called the density matrix whose elements are

$$\rho_{pn} = \langle u_p\,|\,\rho\,|\,u_n\rangle = C_n^* C_p \qquad (8)$$

The density operator ρ derived by

$$\rho = |\,\psi\rangle\langle\psi\,|$$

According to equation (8), equation (4)indicates that the sum of the diagonal elements of the density matrix is equal to 1.

$$\sum_n |\,C_n\,|^2 = \sum_n \rho_{nn} = \mathrm{Tr}\rho = 1 \qquad (9)$$

Trace means the sum of the diagonal terms of matrix element is equal to 1.Now, we can express the radiation field with the density matrix from where the rows and columns are labeled by two basis polarization states $|+\rangle$ and $|-\rangle$ as

$$\rho^{r} = \begin{bmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{bmatrix} = \begin{bmatrix} C_1C_1^* & C_1C_2^* \\ C_2C_1^* & C_2C_2^* \end{bmatrix}$$

$$= \begin{bmatrix} |C_1|^2 & C_1C_2^* \\ C_2C_1^* & |C_2|^2 \end{bmatrix} \tag{10}$$

and the trace of the density matrix is

$$\mathrm{Tr}\rho^{r} = [\rho_{++} + \rho_{--}] = |C_1|^2 + |C_2|^2 = 1 \tag{11}$$

Using equations (3) and (8), equation (4) becomes

$$\langle u_n | A | u_p \rangle = A_{np} \tag{12}$$

$$\rho_{pn} = C_n^* C_p = \langle u_p | \rho | u_n \rangle \tag{13}$$

$$\langle A \rangle = \langle \psi | A | \psi \rangle = \sum_{n,p} C_n^* C_p A_{np} \tag{14}$$

$$\langle A \rangle = \sum_{n,p} \langle u_p | \rho | u_n \rangle \langle u_n | A | u_p \rangle \tag{15}$$

$$\sum_n | u_n \rangle \langle u_n | = 1 \tag{16}$$

$$\langle A \rangle = \sum_p \langle u_p | \rho A | u_p \rangle \tag{17}$$

$$\langle A \rangle = \mathrm{Tr}\,\{\rho A\} \tag{18}$$

Polarization Matrix and Stokes Parameters

The two dimensional Hermitian matrix can be expressed as a linear combination of the unit matrix and Pauli's matrices. So the radiation density matrix can generally be written as

$$\rho^{\gamma} = \frac{1}{2}(1 + \vec{\sigma}.\vec{S}) , \tag{19}$$

where $\vec{\sigma}$ is the vector of which the components are the Pauli's matrices σ_x,$\sigma_y$and σ_z.$\vec{S}$ is a polarization vector of the radiation.

$$\vec{S} = S_x\hat{x} + S_y\hat{y} + S_z\hat{z}$$

$$\vec{\sigma} = \sigma_x\hat{x} + \sigma_y\hat{y} + \sigma_z\hat{z} .$$

Therefore $\rho^{\gamma} = \frac{1}{2}(1 + \sigma_x S_x + \sigma_y S_y + \sigma_z S_z) ,$ (20)

where $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$\rho^{\gamma} = \frac{1}{2}\left[\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}S_x + \begin{bmatrix}0 & -i\\i & 0\end{bmatrix}S_y + \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}S_z\right]$$

$$\rho^{\gamma} = \frac{1}{2}\left[\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + \begin{bmatrix}0 & S_x\\S_x & 0\end{bmatrix} + \begin{bmatrix}0 & -iS_y\\S_y & 0\end{bmatrix} + \begin{bmatrix}S_z & 0\\0 & -S_z\end{bmatrix}\right]$$

$$\rho^{\gamma} = \frac{1}{2}\begin{bmatrix}1+S_z & S_x - iS_y\\S_x + iS_y & 1 - S_z\end{bmatrix}, \tag{21}$$

and the trace of the density matrix is given by

$$\mathrm{Tr}\rho^{\gamma} = \frac{1}{2}(1 + S_z + 1 - S_z) = 1\,. \tag{22}$$

Comparing equation (22) with equation (11), the polarization state of the radiation can be given as follow

The total intensity I is

$$I = \mathrm{Tr}\rho^{\gamma} = \left(\rho_{+,+} + \rho_{-,-}\right). \tag{23}$$

According to the equation (11) the linear polarization is

$$Q = S_x = \mathrm{Tr}(\rho^{\gamma}\sigma_x) = \left(\rho_{+,-} + \rho_{-,+}\right), \tag{24}$$

the plane polarization

$$U = S_y = \mathrm{Tr}(\rho^{\gamma}\sigma_y) = i\left(\rho_{+,-} - \rho_{-,+}\right), \tag{25}$$

and the circular polarization

$$V = S_z = \mathrm{Tr}(\rho^{\gamma}\sigma_z) = \left(\rho_{+,+} - \rho_{-,-}\right), \tag{26}$$

where I, Q, U, V are well-known four Stokes parameters.

Scattering Matrix

If density matrix $\rho(k')$ describes the state of polarization of the incident radiation, the density matrix $\rho(k)$ representing the state of polarization of the scattered radiation is given by

$$\rho(k) = T\rho(k')T^{+}, \tag{27}$$

where T matrix is defined in terms of its elements.

If stokes vectors $S'(k')$ characterizing the state of polarization of the scattered radiation is

$$S(k) = MS'(k'), \tag{28}$$

where, M is the 4 × 4 matrix and is called scattering matrix.

Using the general form of density matrix

$$\rho = \frac{1}{2}\sum_{p=0}^{3}\sigma_p S_p\,, \tag{29}$$

in equation (27) and comparing with equation (28), we obtain scattering matrix elements

$$M_{pp'} = \frac{1}{2}\mathrm{Tr}(T\sigma_{p'}T^{+}\sigma_{p}), \qquad (30)$$

where the matrices σ_p are explicitly given by

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad (31)$$

By using the equation (30), the scattering matrix elements in terms of the polarization states $\mu = \mu' = \pm 1$ is obtained. These scattering matrix elements are shown below:

$$M_{00} = \frac{1}{2}(T_{++}T^{*}_{++} + T_{+-}T^{*}_{+-} + T_{-+}T^{*}_{-+} + T_{--}T^{*}_{--})$$

$$M_{01} = \frac{1}{2}(T_{+-}T^{*}_{++} + T_{++}T^{*}_{+-} + T_{--}T^{*}_{-+} + T_{-+}T^{*}_{--})$$

$$M_{02} = \frac{1}{2}i(T_{+-}T^{*}_{++} - T_{++}T^{*}_{+-} + T_{--}T^{*}_{-+} - T_{-+}T^{*}_{--})$$

$$M_{03} = \frac{1}{2}(T_{++}T^{*}_{++} - T_{+-}T^{*}_{+-} + T_{-+}T^{*}_{-+} - T_{--}T^{*}_{--})$$

$$M_{10} = \frac{1}{2}(T_{++}T^{*}_{-+} + T_{+-}T^{*}_{--} + T_{-+}T^{*}_{++} + T_{--}T^{*}_{+-})$$

$$M_{11} = \frac{1}{2}(T_{+-}T^{*}_{-+} + T_{++}T^{*}_{--} + T_{--}T^{*}_{++} + T_{-+}T^{*}_{+-})$$

$$M_{12} = \frac{1}{2}i(T_{+-}T^{*}_{-+} - T_{++}T^{*}_{--} + T_{--}T^{*}_{++} - T_{-+}T^{*}_{+-})$$

$$M_{13} = \frac{1}{2}(T_{++}T^{*}_{-+} - T_{+-}T^{*}_{--} + T_{-+}T^{*}_{++} - T_{--}T^{*}_{+-})$$

$$M_{20} = \frac{1}{2}i(T_{++}T^{*}_{-+} + T_{+-}T^{*}_{--} - T_{-+}T^{*}_{++} - T_{--}T^{*}_{+-})$$

$$M_{21} = \frac{1}{2}i(T_{+-}T^{*}_{-+} + T_{++}T^{*}_{--} - T_{--}T^{*}_{++} - T_{-+}T^{*}_{+-})$$

$$M_{22} = -\frac{1}{2}(T_{+-}T^{*}_{-+} - T_{++}T^{*}_{--} - T_{--}T^{*}_{++} + T_{-+}T^{*}_{+-})$$

$$M_{23} = \frac{1}{2}i(T_{++}T^{*}_{-+} - T_{+-}T^{*}_{--} - T_{-+}T^{*}_{++} + T_{--}T^{*}_{+-})$$

$$M_{30} = \frac{1}{2}(T_{++}T^{*}_{++} + T_{+-}T^{*}_{+-} - T_{-+}T^{*}_{-+} - T_{--}T^{*}_{--})$$

$$M_{31} = \frac{1}{2}(T_{+-}T^{*}_{++} + T_{++}T^{*}_{+-} - T_{--}T^{*}_{-+} - T_{-+}T^{*}_{--})$$

$$M_{32} = \frac{1}{2}i(T_{+-}T^{*}_{++} - T_{++}T^{*}_{+-} - T_{--}T^{*}_{-+} + T_{-+}T^{*}_{--})$$

$$M_{33} = \frac{1}{2}(T_{++}T^{*}_{++} - T_{+-}T^{*}_{+-} - T_{-+}T^{*}_{-+} + T_{--}T^{*}_{--})$$

Results and Conclusion

Transition Amplitude for $J_i = \frac{1}{2} \to J_n = \frac{3}{2} \to J_f = \frac{1}{2}$

We consider the scattering of polarized radiation by an atom which makes a transition from an initial state $|\psi_i\rangle$ with energy E_i, total angular momentum J_i and parity π_i to a final state $|\psi_f\rangle$ with energy E_f, total angular momentum J_f and parity π_f. The left and right circular polarization states of incident and scattered radiations are denoted by $\mu' = \mu = \pm 1$.

The transition matrix element for scattering of polarized radiation,

$$\langle \phi_f | \rho^f | \phi_{f'} \rangle = \sum_{i,i'} \rho^f \sum_n \langle \phi_f | H_{int} | \phi_n \rangle F_n \langle \phi_n | H_{int} | \phi_i \rangle$$

$$\times \sum_{n'} \langle \phi_{i'} | H^+_{int} | \phi_{n'} \rangle F^*_{n'} \langle \phi_{n'} | H^+_{int} | \phi_{f'} \rangle \qquad (32)$$

Where the frequency dependent profile function $F_n = \dfrac{1}{E_i + \hbar\omega_i - E_n - i\hbar\Gamma_n}$, when we take the basis states $|\phi\rangle$ are the total angular momentum $|jm\rangle$ basis states for atomic scattering process.

Equation (32) becomes

$$\rho^f_{m_f m_{f'}, \mu_f \mu'_f} = \sum_{\mu_i, \mu'_i = \pm 1} \sum_{m_i, m_{i'}} \sum_{m_n, m_{n'}} \rho^i_{\mu_i, \mu'_i} \langle k\mu_f, J_f m_f | H_{int} | J_n m_n \rangle$$

$$\times F_n \langle J_n m_n | H_{int} | k\mu_i \rangle \langle k\mu'_i, J_i m_{i'} | H^+_{int} | J_n m_{n'} \rangle$$

$$\times F^*_{n'} \langle J_n m_{n'} | H^+_{int} | j_f m_{f'}, k\mu'_f \rangle \qquad (33)$$

where ρ^i contains the initial polarization of both atom and radiation. We assume that there is no entanglement between initial states of atom and radiation. It means we can write $\rho^i = \rho^{ir}\rho^{iA}$.

The matrix elements for scattered radiation with polarization μ_f along the direction (θ, ϕ),

$$\langle k\mu_f, J_f m_f | H_{int} | J_n m_n \rangle = E_{m_f m_n}(\mu_f)$$

$$= (2\pi)^{1/2} \sum_{L_e = |J_f - J_n|}^{J_f + J_n} (-i)^{Le} (2L_e + 1)^{1/2}$$

$$\times \langle J_f \| E_{L_e} \| J_n \rangle \sum_{M_e = -L_e}^{L_e} C(J_f, L_e, J_n; m_f, M_e, m_n)$$

$$\times (-i\mu_f)^{h(L_e)} D^{L_e}_{M_e, \mu_f}(\phi, \theta, 0)^* \qquad (34)$$

where $$h(L_e) = \frac{1}{2}[1 + \pi_n \pi_f (-1)^{L_e}].$$

The π_n, π_f are parities of the intermediate state and final state. $\langle J_f \| E_{L_e} \| J_n \rangle$ represents the reduced matrix element, $C(J_f, L_e, J_n; m_f, M_e, m_n)$ is the Clebsch-Gordon coefficient and $D^{L_e}_{M_e, \mu_f}(\phi, \theta, 0)$ is the Winger's D-functions.

The reduced matrix element $\langle J_f \| E_{L_e} \| J_n \rangle$ for emission represents either an electric or a magnetic 2^L pole strength depending on whether h (L) is equal to one or zero.

The matrix element for incident radiation with polarization μ_i along the direction (θ', ϕ') is also given by

$$\langle J_n m_n | H_{int} | J_p m_i, k\mu_i \rangle = A_{m_n m_i}(\mu_f)$$

$$= (2\pi)^{1/2} \sum_{L_e = |J_i - J_n|}^{J_i + J_n} (i)^{L_a} (2L_a + 1)^{1/2}$$

$$\times \langle J_n \| A_{L_a} \| J_i \rangle \sum_{M_a = -L_a}^{L_a} C(J_i, L_a, J_n; m_i, M_a, m_n)$$

$$\times (i\mu_i)^{h(L_a)} D^{L_a}_{M_a, \mu_i}(\phi', \theta', 0) \qquad (35)$$

Similarly, the matrix element for incident radiation with polarization μ_i' is given by

$$\langle k\mu_i', J_i m_{i'} | H^{+}_{int} | J_n m_{n'} \rangle = \langle J_n m_{n'} | H_{int} | k\mu_{i'}, J_i m_{i'} \rangle^* = A_{m_{n'} m_{i'}}(\mu_i')$$

$$= (2\pi)^{1/2} \sum_{L_{a'} = |J_i - J_n|}^{J_i + J_n} (-i)^{L_{a'}} (2L_a + 1)^{1/2}$$

$$\times \langle J_n \| A_{L_{a'}} \| J_i \rangle^* \sum_{M_{a'} = -L_{a'}}^{L_{a'}} C(J_i, L_{a'}, J_n; m_{n'}, M_{a'}, m_{n'})$$

$$\times (-i\mu_i')^{h(L_{a'})} D^{L_{a'}}_{M_{a'}, \mu_i'}(\phi', \theta', 0)^* \qquad (36)$$

and the matrix element for scattered with polarization μ_f is given by

$$\langle J_n m_{n'} | H^{+}_{int} | J_f m_f, k\mu_f \rangle = \langle k\mu_f, J_f m_f | H_{int} | J_n m_{n'} \rangle^* = E_{m_f m_{n'}}(\mu_f')$$

$$= (2\pi)^{1/2} \sum_{L_{e'} = |J_f - J_n|}^{J_f + J_n} (i)^{L_{e'}} (2L_{e'} + 1)^{1/2}$$

$$\times \langle J_f \| E_{L_{e'}} \| J_n \rangle^* \sum_{M_{e'} = -L_{e'}}^{L_{e'}} C(J_f, L_{e'}, J_n; m_f, M_{e'}, m_{n'})$$

$$\times (i\mu_f')^{h(L_{e'})} D^{L_{e'}}_{M_{e'}, \mu_f'}(\phi, \theta, 0) \qquad (37)$$

Therefore, equation (33) becomes

$$\rho^{f}_{m_f m_{f'}, \mu_f \mu_f'} = \sum_{\mu_i, \mu_i' = \pm 1} \sum_{m_i, m_{i'}} \sum_{m_n, m_{n'}} \rho^{i}_{\mu_i, \mu_i'} (2\pi)^2 F_{m_n} F^*_{m_{n'}}$$

$$\times \sum_{L_e = |J_f - J_n|}^{J_f + J_n} (-i)^{L_e} (2L_e + 1)^{1/2} \langle J_f \| E_{L_e} \| J_n \rangle$$

$$\times \sum_{M_e = -L_e}^{L_e} C(J_f, L_e, J_n; m_f, M_e, m_n)(-i\mu_f)^{h(L_e)} D^{L_e}_{M_e, \mu_f}(\phi, \theta, 0)^*$$

$$\times \sum_{L_a = |J_i - J_n|}^{J_i + J_n} (i)^{L_a} (2L_a + 1)^{1/2} \langle J_n \| A_{L_a} \| J_i \rangle$$

$$\times \sum_{M_a=-L_a}^{L_a} C(J_i, L_a, J_n; m_i, M_a, m_n)(-i\mu_i)^{h(L_a)} D^{L_a}_{M_a,\mu_i}(\phi', \theta', 0)$$

$$\times \sum_{L_{a'}=|J_i-J_n|}^{J_i+J_n} (-i)^{L_{a'}} (2L_a+1)^{\frac{1}{2}} \langle J_n \| A_{L_{a'}} \| J_i \rangle^*$$

$$\times \sum_{M_{a'}=-L_{a'}}^{L_{a'}} C(J_i, L_{a'}, J_n; m_{i'}, M_{a'}, m_{n'})(i\mu_i')^{h(L_{a'})} D^{L_{a'}}_{M_{a'},\mu_i'}(\phi', \theta', 0)^*$$

$$\times \sum_{L_{e'}=|J_f-J_n|}^{J_f+J_n} (i)^{L_{e'}} (2L_{e'}+1)^{\frac{1}{2}} \langle J_f \| E_{L_{e'}} \| J_n \rangle^*$$

$$\times \sum_{M_{e'}=-L_{e'}}^{L_{e'}} C(J_f, L_{e'}, J_n; m_f, M_{e'}, m_{n'})(i\mu_f')^{h(L_{e'})} D^{L_{e'}}_{M_{e'},\mu_f'}(\phi, \theta, 0) \qquad (38)$$

Since no observation are made on the state of polarization of the atom in the individual final state, the polarization states of the scattered radiation may be defined through a density matrix ρ^f as

$$\rho^f_{\mu_f\mu_{f'}} = \sum_{m_f} \rho^f_{m_f m_{f'};\mu_f\mu_{f'}}$$

$$\rho^f_{\mu_f\mu_{f'}} = \rho^i 36(\pi)^2 \left|\langle\frac{1}{2} \| E_1 \| \frac{3}{2}\rangle\right|^2 \left|\langle\frac{3}{2} \| A_1 \| \frac{1}{2}\rangle\right|^2 F_{m_n} F_{m_{n'}}{}^*$$

$$\sum_{m_f=-\frac{1}{2}}^{\frac{1}{2}} \sum_{m_i=-\frac{1}{2}}^{\frac{1}{2}} \sum_{\mu_i=-1}^{1} \sum_{m_n=-\frac{3}{2}}^{\frac{3}{2}} \sum_{M_e=-1}^{1} C\left(\frac{1}{2}, 1, \frac{3}{2}; m_f, M_e, m_n\right) \mu_f D^1_{M_e,\mu_f}(\phi, \theta, 0)^*$$

$$\sum_{M_a=-1}^{1} C\left(\frac{1}{2}, 1, \frac{3}{2}; m_i, M_a, m_n\right) \mu_i D^1_{M_a,\mu_i}(\phi', \theta', 0)$$

$$\sum_{m_{i'}=-\frac{1}{2}}^{\frac{1}{2}} \sum_{m_{n'}=-\frac{3}{2}}^{\frac{3}{2}} \sum_{\mu_{i'}=-1}^{1} \sum_{M_{a'}=-1}^{1} C\left(\frac{1}{2}, 1, \frac{3}{2}; m_{i'}, M_{a'}, m_{n'}\right) \mu_{i'} D^1_{M_{a'},\mu_{i'}}(\phi', \theta', 0)^*$$

$$\sum_{M_{e'}=-1}^{1} C\left(\frac{1}{2}, 1, \frac{3}{2}; m_f, M_{e'}, m_{n'}\right) \mu_{f'} D^1_{M_{e'},\mu_{f'}}(\phi, \theta, 0) \qquad (39)$$

The characteristics of the incident radiation(ϕ', θ') and scatteredradiation$(\phi, \theta)$are shown in figure (1). We calculate the degree of linear polarization of scattered radiation in the absence of an external magnetic field to check our theoretical calculations and algorithm of our numerical program. We first calculate the degree of linear polarization of scattered radiation in the weak magnetic field when the incident radiation is linearly polarized for some chosen angles. These results are shown in Table (1). For unpolarized case, the incident radiation is 100% linearly polarized and its direction is taken as $\theta' = 90^o$, $\phi' = 0^o$. The scattered radiation is assumed to be $\theta = 90^o$, $\phi = 90^o$. The incident radiation is $\theta' = 30^o$, $\phi' = 0^o$ and the scattered radiation is $\theta = 70^o$, $\phi = 30^o$.These results are presented in Table (2).The comparison of the degree of linear polarization of the scattered radiation is shown in Table (3).

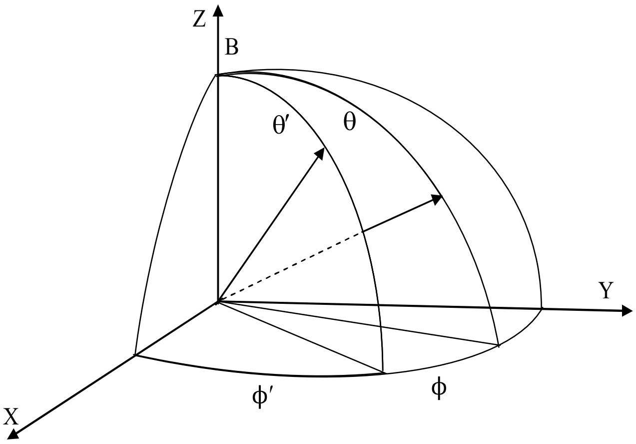

Figure 1 Angular characteristics of the incident (ϕ', θ') and scattered (ϕ, θ) radiation.

Table 1 The result of polarization when the incident radiation is linearly polarized for some chosen angles.

Type of Incident Radiation	Incident Angles		Scattered Angles		Linear Polarization of Scattered Radiation (percent) (Calculated Results)
	θ'	ϕ'	θ	ϕ	
Linearly polarized	20°	0°	50°	20°	56
	30°	0°	70°	30°	35.85
	40°	0°	60°	30°	52
	60°	20°	80°	30°	55
	90°	0°	90°	90°	60
	100°	30°	80°	30°	65

Table 2 The result of polarization when the incident radiation is unpolarized for some chosen angles.

Type of Incident Radiation	Incident Angles		Scattered Angles		Linear Polarization of Scattered Radiation (percent)
	θ'	ϕ'	θ	ϕ	
Unpolarized	90°	0°	90°	90°	43
	30°	0°	70°	30°	−13

Table 3 Linear Polarization (percent) of $J = \frac{1}{2} \rightarrow \frac{3}{2} \rightarrow \frac{1}{2}$ scattered radiation for $\theta' = 90^o$, $\phi' = 0^o$, $\theta = 90^o$, $\phi = 90^o$ and $\theta' = 30^o$, $\phi' = 0^o$, $\theta = 70^o$, $\phi = 30^o$

Calculated result (percent)	Theoretical result (percent)	Experimental result (percent)
60	60	60
35.85	35.53	36

Conclusion

We have computed the polarization of scattered line radiation from the upper level with angular momentum $J_u = \frac{3}{2}$ to a lower level with angular momentum $J_\ell = \frac{1}{2}$ transition in the absence of magnetic fields. The Stokes parameters of the incident radiation were taken to be unpolarized and there is no lower level polarization of an atom. We calculate the simplet geometrical arrangement of scattering in which the incident radiation is 100% linearly polarized and its direction is taken as $\theta' = 90^\circ$, $\phi' = 0^\circ$. The scattered radiation is assumed to be $\theta = 90^\circ$, $\phi = 90^\circ$.For unpolarized case incident radiation is $\theta' = 30^\circ$, $\phi' = 0^\circ$ and the scattered radiation is $\theta = 70^\circ$, $\phi = 30^\circ$.We particularly choose this set of parameters to compare our calculated result with well-established theoretical result as well as experimental result. The comparison of the degree of linear polarization of the scattered radiation is given in Table (3). In this case we obtain the good agreement with theoretical results as well as experimental results.

Acknowledgements

We wish to express our deepest gratitude to Dr. Ba Han, Rector, Meiktila University and Dr, Kay Thi Thin Pro-Rector, Meiktila University for their kind permission to do this work. We would like to express sincere thanks to Professor Dr. Hla Hla Than, Head of Department of Physics, Meiktila University and Dr. Daw Hla Wynn, Professor , Department of Physics, Meiktila University for their permission and moral support throughout the paper. We are also grateful to all teaching staff of Physics Department, Meiktila University.

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